Systems of Equations

Substitution, elimination, and graphing

Systems of equations is the term used to describe solving for several variables using several equations. The key to solving systems of equations is to:

- Ensure you have the same number or more equations than the number of unknown variables you have (e.g. three unknowns need at least three equations to solve, four unknowns needs four equations, etc.).
- All of your unknown variables are in at least one of your equations.
- Be open to using different/multiple methods to solve the system.

Substitution

Step 1: Isolate one variable in one equation.

Solve: $3x + y = 7$
$2x - 2y = 2$

$3x + y = 7$
$-3x \quad -3x$
$y = 7 - 3x$

Step 2: Plug in isolated variable into other equation and solve.

$2x - 2(7 - 3x) = 2$
$2x - 14 + 6x = 2$
$+14 \quad +14$
$2x + 6x = 16$
$\frac{8x}{8} = \frac{16}{8} \Rightarrow x = 2$

Step 3: Plug known variable into equation to solve for the unknown variable.

$3(2) + y = 7$
$6 + y = 7$
$-6 \quad -6$
$y = 1$

Elimination

Step 1: Manipulate equations so the x or y coefficients are equal but opposite.

Solve: $5y - 3x = 16$
$4y + 2x = 26$

$2(5y - 3x) = 2(16)$
$3(4y + 2x) = 3(26)$
$10y - 6x = 32$
$12y + 6x = 78$

x coefficients are equal and opposite

Step 2: Add equations together and solve for variable.

$10y - 6x = 32$
$+ \; 12y + 6x = 78$
$\frac{22y}{22} = \frac{110}{22} \quad y = 5$

Step 3: Plug known variable into equation to solve for the unknown variable.

$5(5) - 3x = 16$
$25 - 3x = 16$
$-25 \quad -25$
$\frac{-3x}{-3} = \frac{-9}{-3} \quad x = 3$

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Graphing

Solve: $x + y = 5$ $2y - x = 1$

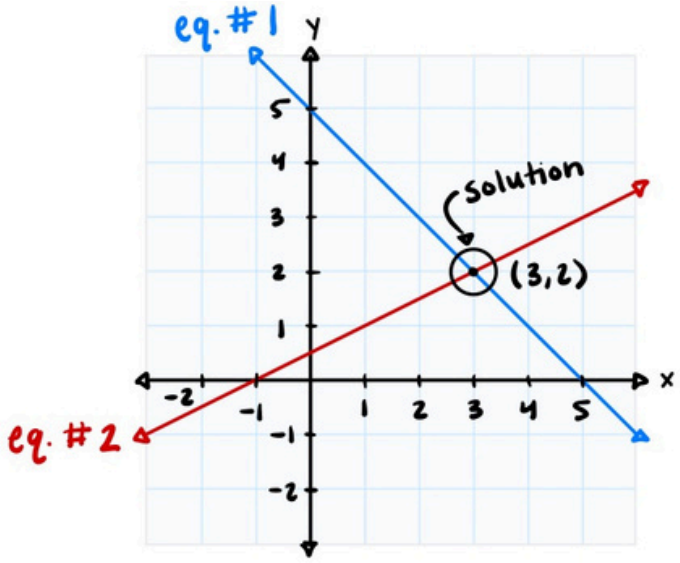
Step 1: Put equations into y = mx + b form. (m = slope and b = y-intercept).

1) $x + y = 5 \longrightarrow y = -x + 5$

2) $2y - x = 1 \longrightarrow y = \frac{1}{2}x + \frac{1}{2}$

Step 2: Graph both equations on x-y axis (Desmos is a great tool for this).

See graph. →



Step 3: Find where the lines intercept (this can be multiple places).

Intercept: (3, 2) → x = 3, y = 2

Step 4: Interpret your results. What does the intercept(s) mean in context?

Now you try!

1) **Solve:** $7x + y = -10$ $-2y - 6x = -4$

1) **Solution:** x = -3 y = 11

2) **Solve:** $x*\cos(40°) - y*\cos(70°) = 0$
$x*\sin(40°) + y*\sin(70°) - 50 = 0$
* = multiplication

2) **Solution:** x ≈ 18.2
y ≈ 40.76

3) **Solve:** $x + 2y - 3z = -25$
$-7y - 5x + z = -4$
$-0.5x + 2z - 9y = 25$

3) **Solution:** x = 4
y = -1
z = 9